International Journal of Advanced Research in Engineering Technology & Science

ISSN: 2349-2819

Email: editor@ijarets.org Volume-8, Issue-9 September – 2021 www.ijarets.org

EXTREMAL SETS AND MIN-MAX PROPERTIES IN TOPOLOGICAL SPACES

Ms. Himanshu Kumari,
Research Scholar, Department of Mathematics,
Monad University.

Dr. R. B. Singh,
Supervisor, Professor & Head, Department of Mathematics,
Monad University, Hapur

1.1 Introduction

The study of an object's topological qualities is similar to the study of its geometric properties, which is congruence, in geometry. If a property remains unchanged when subjected to a homeomorphism, we say that it is a topological property. Any bijection f;X→ Y where both f and f^{-1} are continuous is considered a homeomorphism from space X to space Y. When this kind of correspondence holds, we say that X is homeomorphic to Y.

If X is a non-empty set and ℑ is a class of subjects of X, known as a topology on X, and ℑ meets the following characteristics, then the space (X, ℑ) is termed a topological space.

1. $\phi \in ℑ$ and $X \in ℑ$
2. A set ℑ in is the union of all classes of sets in ℑ.
3. Any finite class of sets ℑ in has an intersection with another set in ℑ.

In order to be considered a topological space, a metric space must meet certain criteria. Metrics, then, produce topologies. For every set X, there exists a non- negative real-valued function p on the product of X and X that is defined as a quasi pseudo meter.

1. For any $x \in X$, p(x, x) = 0.
2. where p(x, z) is less than or equal to P(x, y) plus P(y, z) for all x and y in X. The conjugate

quasi-pseudometric q of every quasi-psudometric p is defined as q(x,y)
= p(y,x) for every x, $y \in X$. If a non-empty set X is used to construct a structure (x,p,q)that simultaneously takes two metrics p and q. So, Kelly used bitopological spaces to investigate the consequences of utilizing the topology described by these two distancefunctions p and q concurrently.

He established the concept of a bitopological space as X endowed with two topologiesof his choosing.

The application of topological methods to differentiate between separate points and disjoint sets is the subject of the separation axioms. We may desire elements of a topological space to be topologically distinguishable, in addition to being distinct.

Likewise, it's not sufficient for topological space subsets to be disjoint; we may need their separation in some other manner as well. As a general rule, the separation axioms state that if two sets or points can be distinguished in a weak sense, then they must also be distinguished in a stronger meaning.

If, for every pair of unique points x and y in X, there exists an open set that contains only one of these points and not the other, then we say that X is a T_0-space (or that thetopology on X is T_0).

For every two points x and y in a topological space X, we say that X is a T_1-space if and only if their respective neighborhoods do not contain one other. For every two points x and y in a space X, we say that space is a T_2-space (Hausdroff space) if and only if there are two separate open sets U and V in X such that $x \in U$ and $y \in V$.

If for every closed set A in X there exists an open set $x \notin A$, then for every set U and V there exists an $A \uplus V$, then topological space X is a regular space.

When A and B are two distinct closed sets in X, then there are two distinct open setsU and V such that $A \subset U$ and $B \subset V$, and topological space X is normal iff this holds. Typical T_1-space is going to be referred to as T_4-space.

$(X, \mathfrak{I}_1, \mathfrak{I}_2)$ is the ordered triple of a non-empty set X and two arbitrary topologies; a bitopological space is a set that contains both types of topologies. The two topologies can be chosen at random.

Studies of topology have traditionally focused on its sets endowed with certain attributes. Open sets in their various forms have been the most common. Since the topology was introduced on a non-empty set, thousands of mathematicians have foundnew types of open sets. The emergence of new classes of sets has long piqued the curiosity of several mathematicians due to the fact that certain sets exhibited peculiar behavior. Nakaoka and Oda [18] - [20] established the notions of minimal open sets, maximal open sets, minimal closed sets, and maximal closed sets, among other invariants of open sets that have been introduced from time to time. If there is no suitable open subset of x that can be sandwiched between ϕ and U in a topological space $(X, \mathfrak{I})$, then U is referred to as minimally open. The same is true for appropriatenonempty open sets of X; when U and X are the only open supersets of U, we say thatU is maximally open. A topological space can also have minimal and maximal closed sets defined in a comparable way.

The concept of open sets can be applied broadly to every element of a set X, whetherit lies inside or outside of a subset of X, and many properties of set theory and topological spaces can be exhibited on such sets because of their commonalities. We developed a novel class of sets defined on a topological space and studied its behaviorunder various conditions and suppositions as a result of all these events. The class of extremal sets is what we call this new category of sets, and we investigate its characteristics through the lenses of subsets, supersets, intersection, and union of sets. In most cases, it has been discovered that open sets and extremal sets are completely separate.

New descriptions of sets emerge, however, when both traits are present, i.e., when a set is open and extremal. Research on these traits has also taken place in this location. Several instances have also been provided to back up these traits.

We go into further detail about extremal sets and how they behave and what their characteristics are when minmax attributes are present in the section that follows. Additionally, we handle these sets in conjunction with linked topological spaces and explore the unique behavior of certain features of extremal sets.

1.2 Review of Literature

Awad, Ghufran & Jumaili, Alaa. (2021). Utilizing another generalized open set, E- open, this paper introduces and studies a new concept of separated sets called E- separated sets. It also introduces a new class of connected spaces called strongly Д-connected spaces and investigates some basic properties of these spaces. Various descriptions and basic characteristics of these types of linked spaces with certain E- Separation axioms and compact spaces are derived. The discussion is on the actions ofE-connected spaces in relation to several popular mappings. In addition, we build novel topological spaces on a graph that is connected. Classification of Mathematics: 54B05, 54B10, 54C10, 54D18, 90D42.

Nachimuthu, Murugavalli & Pushpalatha, A. (2021). In this work we present the novel concept of gλ-irresolute mappings, gλ-continuous, gλ-closed sets, gλ-open sets, gλ-homeomorphisms in topological space. Additionally, we present two additional spaces, T N-space and -space. In topological spaces, we investigate a few of its characteristics. The initial step of this research is to present a novel kind of closed map, the λ^{Λ}-closed map. Additionally, we present a separate category of homeomorphisms known as λΛ - Homeomorphisms, which are less robust than homeomorphisms. We further prove that the set of all λ^{Λ} - Homeomorphisms forms agroup under the operation of composition of maps and introduce $* \Lambda^{\lambda}$ - Homeomorphisms. Classes in mathematics for the year 2000: 54C08 and 54D05.

Zahan, Ishrat & Nasrin, Rehena. (2021). Fuzzy sets have tremendous topological applications. This article can help mathematicians become more aware of these topological applications on fuzzy sets. This study begins with a classification of fuzzysets and topological spaces, followed by an analysis of the relationships between their constituent parts. To make things easier from a mathematical perspective, we have forgotten the most fundamental definitions of crisp set and fuzzy set. After that, we covered the topic of topological spaces. In the last part, we have established a relationship between fuzzy sets and topological spaces; these spaces constitute our primary focus. In addition, the piece has been wrapped up by looking at its properties and how these places are closed off in relation to one other.

Beg, Ismat et al., (2021). Our proposal is an extension of S-metric spaces called S^{JS}-metric spaces. We then conduct research into S^{JS}-metric spaces and demonstrate multiple theorems. In this context, we accomplish various classical results, such as Cantor's intersection theorem, and we also handle abstract S^{JS}- topological spaces generated by S^{JS}-metric.

Khalaf, Alias et al., (2020) We examine the basic characteristics, connections, and descriptions of three kinds of separation axioms in this work, which are introduced through ΰ-open sets: ΰ-regular, entirely ΰ-regular, and ΰ-normal space. The famous Lemma and Tietze Extension Theorem of Urysohn and Tietze are extended to spaces that are ω-normal. We refine some established outcomes. Additionally, ΰ-open sets are used to study additional generalized ideas.

Karthiksankar, P. & Subbulakshmi, Mrs. (2020). The primary goal of this article is to present a novel topological space (the "Tiny topological space") and to explore the characteristics of generalized closed sets within it.

Bhardwaj, Nitin & Aaliya, Mir. (2020). A novel subclass of generalized closed sets, between closed sets and weakly generalized closed sets, is defined in this study as pre-weakly generalized closed sets in topological space. Additionally, we explore and analyze their basic characteristics and apply the findings to other specified categories of generalized closed sets.

1.3 Objectives of the study:-

Objective 1 To study separation axioms in topological and bitopological spaces analytically.

Objective 2 To examine the worth of existing separation axioms in topological and bitopological space.

1.4 Extremal Sets

Definition 5.2.1 The definition of an extremal subset of X in a topological space (X, ℑ) is a proper subset B ⊆ X that is open and contains both a and x, where ∀ a ∈ A and ∀ x ∈ (X – A)

Situation 5.2.2 According to the standard topology, every subset of X is a set of extremals.

Case 5.2.3 Every subset of X is an extremal set in a topological space (X, ℑ) with discrete topology.

Situation 5.2.4 Nothing in the set X can be considered an extremal set if the topology of the space (X, ℑ) is indiscrete.

A valid open set {t, p, q} contains both p and q in (X, ℑ), as shown in Example 5.2.5, where t is the point of inclusion in a topological space (X, ℑ). Similarly, for any set P ⊆ X, p in P and q in (X - P), we have an extremal set in (X, ℑ).

The 5.2.6 example No set P ⊆ X will be an extremal set in a set X with a point exclusion topology, where t is the point of exclusion, as shown beneath:

If t is a proper open set in P, then no proper open set in (X, ℑ) can contain both t andq for each q in (X - P). Likewise, t ∈ (X - P) holds if and only if t is not in P. Once again, no suitable open set in (X, ℑ) contains t and q for each q ∈ P.

Example 5.2.7 Take X = {a, b, c, d} as an example and consider a topology on X as

(i) ℑ1 = {ϕ, X, {a}, {b}, {a, b}, {a, b, c}}. Since the element d is not contained in anyopen set under this topology, no subset of X can be considered an extremal set.

(ii) The set ℑ2 is equal to {ϕ, X, {a}, {b}, {a, b}, {a, b, c}, {a, b, d}}. Consider the following set of sets: A1, A2, A3, and A6 are extremal subsets of X, whereas A4, A5, and A7 are not. This is because A1 = {a, b}, A2 = {a}, A3 = {b}, A4 = {c}, A5 = {d},A6 = {c, d}, and A7 = {a, c}.

Corollory 5.2.8 An extremal point is defined as a point x ∈ X for which the set {x} is an extremal set.

The set of real numbers on which the ordinary topology is defined is called a topological space, as shown in Example 5.2.9. Any point on the X-axis is an extremal point in this space.

Instance 5.2.10 All of the points in the set X are extremal points in example 5.2.3.No point of X in example 5.2.4 is an extremal point, as shown in example 5.2.11.
The set {p, a, x} is a proper open set in X that contains both a and x, with p being the point of inclusion. In point inclusion topology, every point a ∈X is an extremal point, just as ∀ x ∈ X.shown in example 5.2.12

No point on X may be an extremal point in point exclusion topology (Exhibit 5.2.13), since no open set contains a point of exclusion.

In example 5.2.7(ii), points a and b are extremal points, however points c and d are not (Exhibit 5.2.14).

Proposition 5.2.15: In a topological space (X, ℑ) where P is an extremal set, (X - P) is also an extremal set.

The evidence. The equation Q = (X - P) can be rewritten as follows: p ∈ Q and x ∈ X

- Q = P. Then, there is an open set that contains p and x, because P is extremal. Since both p and x might be anything, the extremal value of Q = (X - P) follows.

1.5 Topology: Topology Corresponding to a Topological Space Induced byExtremal Sets

Inducing a novel topology on a given topological space is made possible by the collection of extremal sets on that space, which displays various intriguing properties.Following this, we will go over some of the most salient features of the set of extremesets:

Assumption 5.3.1 In a topological space (X, ℑ), the arbitrary intersection of extremal sets is either empty or a set of extremals.

The evidence. The set {Pi | i ∈ Δ} represents an arbitrary collection of extreme subsets of X. Consider P as the absolute value of all i ∈ Pi. We are finished if P equals ϕ. Make P not equal to ϕ. We need to prove that for every element in P and every element outside of P, there exists an open set that contains both of these components for P to be an extremal set as well. Assume that p is an element of P, thenp is an element of p ∈ Pi ∀ i ∈ Δ. For every i ∈ , if x is a member of P, then x is alsoa member of (⋂i ∈ Δ Pi), hence x is a member of Pi. A free subset U of X that contains both p and x exists since p is a member of Pi and x is a member of Pi. Therefore, P is a maximal subset of X as well.

Subsequent 5.3.2 Either an extremal set or an empty space is formed by the finite intersection of extremal sets in the space (X, ℑ).

Statement 5.3.3 In a space (X, ℑ), the union of any two extremal sets can be either another extremal set or the complete set. X.

The evidence. A topological space (X, ℑ) can have an arbitrarily large collection of extremal subsets denoted as {Pi | i ∈ Δ} .Then, P is equal to the product of all i and Pi. We can conclude if P equals X. Show that P is not equal to X. If we want to prove that P is an extremal set as well, we need to prove that there is an open set that contains both the items in P and the elements outside of it. Then, for any i ∈ Δ, p belongs to Pi. If x is a member of P, then it is also a member of (⋃i ∈ ΔPi), which means that x is Pi ∀ i ∈ Δ. Since p is a prime number and x is a prime number, thereis always an open subset U of X that contains both of these elements. P is an extremal subset of X as a result.

Conclusion 5.3.4 Either an extremal set or the entire set is what the union of two extreme sets in a topological space (X, ℑ) represents.

We can conclude that the family of extremal sets in any topological space is closed under both finite intersection and arbitrary union. Therefore, if we think about the set
ℜ= {ϕ, X, E | E is an extremal subset of X}. Based on the theorems stated above, it is clear that this collection may also be expressed as a topology on the topological space (X, ℑ) with the formula

1.5.1 ϕ, X ∈ ℜ.

1.5.2 The arbitrarily combined portions of ℜ are themselves components of ℜ.

1.5.3 The intersection of finite sets of elements is itself a set of elements.

Therefore, for each given topological space, we can create a new topology by collecting extremal sets. The topological space (X, ℜ) that corresponds to this topology, which we will call E-topology or Extremal topology, is hence called Etopological space. In addition, when P is either the empty set ϕ, the entire set X, or an extremal set in X, we say that P is E-open. In the same vein, if P is an element of
ℜ, meaning that P is open in the appropriate E-topology, then P ⊆ X is E-open. Be that as it may, an open set is not necessarily an extremal set. Plus, there's no guaranteethat an extremal set is also an open set. Therefore, in most cases, these two topologies
are not dependent on one another. Numerous exceptional features are displayed by thetopology consisting of extremal sets. Here we will get into a few of them:

In 5.3.5, we find On a topological space (X, ℑ), the clopen elements of E-topology arecreated.

The evidence. It has been demonstrated that this set is closed for both finite and arbitrary intersections. The external sets in a topological space are E-open and E- closed, and since the complement of every extremal set is likewise an extremal set, wesay that the elements of E-topology are clopen.

1.6 Conclusion

Objective 1 To study separation axioms in topological and bitopological spaces analytically.

An interesting new area of research in the challenging subject of topology is the study of separation axioms in topological and bitopological spaces. This analytical study investigates the foundational mechanisms that define the spatial interactions between points in an effort to simplify separation axioms. The goal of this exercise is to highlight the various topological properties that allow spaces to be classified. Thegoal of our exploration of the complex worlds of bitopological spaces and classic topological spaces is to discover the basic laws that govern the isolatability and uniqueness of points in these mathematical domains. We will have to go through the theoretical terrain of both of these places to do this. Because it improves our understanding of abstract spaces and paves the path for practical applications in many other disciplines, studying separation axioms is an interesting and rewarding intellectual endeavor. Our understanding of abstract spaces has improved because to this investigation.

Objective 2 To examine the worth of existing separation axioms in topological and bitopological space.

The goal of this study is to evaluate the usefulness of separation axioms in topological and bitopological space domains, which goes to the heart of mathematical structure and abstraction. Examining the currently used separation axioms is crucial for understanding the fundamental principles that control the spatial interactions inside these different locations. As we navigate the complex landscape of topology and bitopology, we will focus on both the theoretical perfection of these ideas and their practical implications. This study will examine the relevance and utility of well-known separation axioms in order to provide light on their potential application in solving real-world problems and enhancing our comprehension of spatial arrangements. We will go on a journey to better understand mathematics and the real- world consequences of abstract concepts in topological and bitopological spaces as we work to ascertain the validity of these axioms.

REFERENCES

1. D. J. Sharma and B. C. Tripathy, “Pairwise generalized b-R0-spaces in bitopological spaces”, Proyecciones Journal of Mathematics, 36(4), 589-600, 2017.
2. D. J. Sharma, “Weakly b-open functions in bitopological spaces”, Boletim da Sociedade Paranaense de Matematica, 35(2), 105-114, 2017.
3. D. Ray and R. Bhowmik, “On gij closed bi-generalized topological space”,
4. D. S. Jankovic, “On some separation axioms and -closure”, Matem□tički Vesnik, 32(4), 439-449, 1980.
5. E. Ekici, “On δ-preopen sets”, Mathematica, 47(2), 157-164, 2005.
6. E. Ekici, S. Jafari and R. M. Latif, “On a finer topological space than and □ɵ some maps”, Italian Journal of Pure and Applied Mathematics, 27, 293-304,2010.

7. “A Note on Extremally Ω-Disconnectedness”, presented at National Seminar onRecent Advances in Mathematics and Computational Aspects, S.M.P.G.G.P.G. College, Madhavpuram, Meerut (U.P.) in November, 2016.

8. “Extremal Sets with Min Max Properties”, presented at MTMI International Conference on Emerging Issues on Business, Technology and Applied Sciences, Dubai in December 2017.

9. “Ω-Compact Spaces”, presented at International Conference on Frontiers of Science and Technology-2017, held at KIET Group of Institutions, Ghaziabad,
U.P. in January, 2017.

10. 18. Sharma , V.K. g- separation axioms. , Acta Ciencia Indica, vol. XXXIIIM, No.4, 1271 (2007).
11. B. Khalaf, H. M. Darwesh, K. Kannan, “Some types of separation axioms in topological spaces”, Tamsui Oxford Journal of Information and Mathematical Sciences, 28(3), 303-326, 2012.

12. Banaschewski, J. L. Frith and C. R. A. Gilmour, “On the congruence lattice of aframe”, Pacific Journal of Mathematics, 130(2), 209-213, 1987.

13. Bayhan, sadık & Doğan, Çoker. (2021). On separation axioms in intuitionistic topological spaces. International Journal of Mathematics and Mathematical Sciences. 27. 10.1155/S0161171201004884.

14. Bhamini, M.P. The role of semi-open sets in general topology Ph.D. thesis, University of Delhi (1983).
15. C. Carpintero, J. Pacheco, N. Rajesh and E. Rosas, “Properties of nearly continuous multifuctions”, Acta Universitatis Sapientiae Mathematica, 9(1), 13-25, 2017